Algebra 1

$f(x) = ax^2 + c$

Our Goal: To learn to graph quadratic functions of the form $f(x) = ax^2 + bx + c$

Warm Up: Inequalities review

Today's Homework
8.3 Exercises, p.436-437: 4-30 (evens)

Previous Homework
8.2 Exercises, p.429: 4-28 (evens)
use the TI-84 for graphing (graph paper online, if needed)

$y = \frac{9}{2}x^2$

$f(x) = 4x^2 - 5$

$g(x) = f(x) + 7$

$g(x) = 4x^2 - 5 + 7$

$g(x) = 4x^2 + 2$ ←

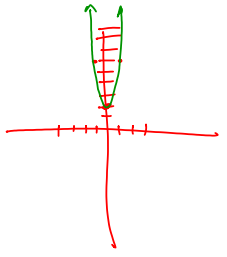
$V(0, 2)$

up.

$y = x^2$

Range $y \geq 2$

decreasing $x < 0$



Complete the exercise.

1. Does (4, 3) satisfy the equation $y = 3x^2 - x + 7$?

2. Does (0, −1) satisfy the equation $y = -2x^2 + \frac{1}{2}x - 1$?

3. Does (5, 0) satisfy the equation $y = 4x^2 - 2x + 4$?

4. Does (−1, −9) satisfy the equation $y = -2x^2 + 3x - 4$?

Solve the inequality. Graph the solution.

1. $4y \geq -12$

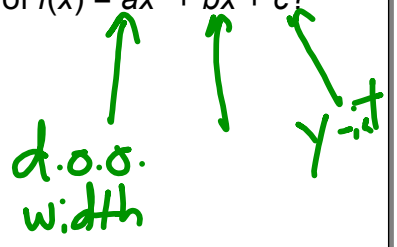
2. $36 > 6t$

3. $\frac{a}{2} > 9.3$

4. $-18 \geq \frac{9}{2}t$

Essential Question

How can you find the vertex of the graph of $f(x) = ax^2 + bx + c$?



Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Work with a partner.

a. Sketch the graphs of $y = 2x^2 - 8x$ and $y = 2x^2 - 8x + 6$.

b. What do you notice about the x-coordinate of the vertex of each graph?

c. Use the graph of $y = 2x^2 - 8x$ to find its x-intercepts. Verify your answer by solving $0 = 2x^2 - 8x$.

d. Compare the value of the x-coordinate of the vertex with the values of the x-intercepts.

Work with a partner.

a. Solve $0 = ax^2 + bx$ for x by factoring.

b. What are the x-intercepts of the graph of $y = ax^2 + bx$?

c. Copy and complete the table to verify your answer.

x	$y = ax^2 + bx$
0	
$-\frac{b}{a}$	

Work with a partner. Complete the following logical argument.

The x-intercepts of the graph of $y = ax^2 + bx$ are 0 and $-\frac{b}{a}$.

The vertex of the graph of $y = ax^2 + bx$ occurs when $x =$ _____.

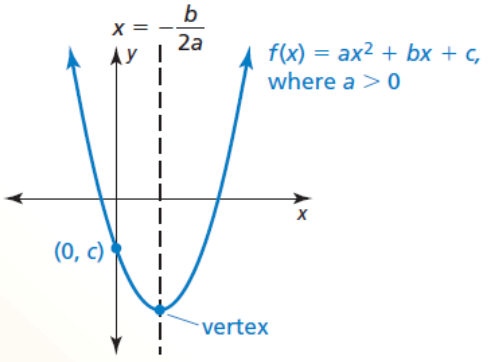
The vertices of the graphs of $y = ax^2 + bx$ and $y = ax^2 + bx + c$ have the same x-coordinate.

The vertex of the graph of $y = ax^2 + bx + c$ occurs when $x =$ _____.

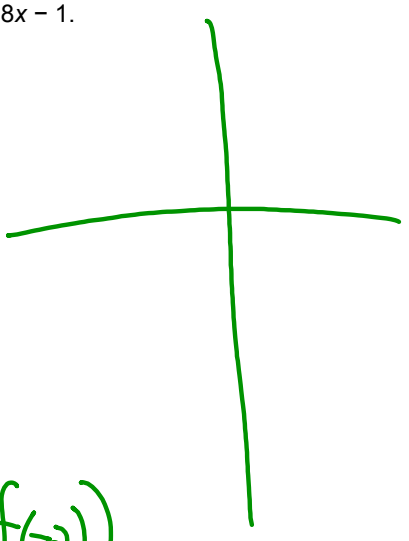
Core Concept

Graphing $f(x) = ax^2 + bx + c$

- The graph opens up when $a > 0$, and the graph opens down when $a < 0$.
- The y-intercept is c.
- The x-coordinate of the vertex is $-\frac{b}{2a}$.
- The axis of symmetry is $x = -\frac{b}{2a}$.



Find (a) the axis of symmetry and (b) the vertex of the graph of $f(x) = 2x^2 + 8x - 1$.



$a = 2$
$b = 8$
-8
4
$-\frac{2}{1}$ $\boxed{-2}$

$(-2, f(-2))$
$(-2, -9)$

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Find (a) the axis of symmetry and (b) the vertex of the graph of the function.

1. $f(x) = 3x^2 - 2x$ **2.** $g(x) = x^2 + 6x + 5$

$\frac{1}{3}$ $\frac{2}{6}$

3. $h(x) = -\frac{1}{2}x^2 + 7x - 4$

$a=3$
$b=-2$

$\frac{-b}{2a}$ $\frac{-(-2)}{2(3)}$

$f(1/3)=3(1/3)^2-2\cdot 1/3$

$f(1/3)=1/3-2/3$

$f(1/3)=-1/3$

$V:\left(\frac{1}{3}, -\frac{1}{3}\right)$

zeros $f(x)=3x^2-2x$

$0=3x^2-2x$

$0=x(3x-2)$

$x=0, \frac{2}{3}$

$h(x)=-\frac{1}{2}x^2+7x-4$

$\left(\frac{-b}{2a}, \right)$ $a=-1/2$
$b=7$

$\left(\frac{-7}{2(\frac{-1}{2})}, \right)$

$\left(7, -\frac{1}{2}(7)^2+7(7)-4\right)$

$-\frac{1}{2}(49)+49-4$

$\frac{49}{2}-\frac{8}{2}$

$\left(7, \frac{41}{2}\right)$

Graph $f(x) = 3x^2 - 6x + 5$. Describe the domain and range.

Graph the function. Describe the domain and range.

4. $h(x) = 2x^2 + 4x + 1$

5. $k(x) = x^2 - 8x + 7$

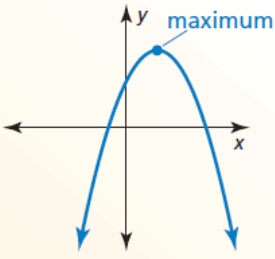
6. $p(x) = -5x^2 - 10x - 2$

Core Concept

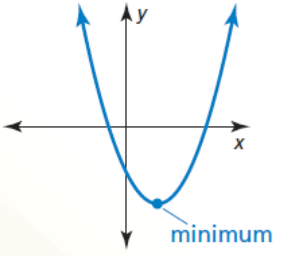
Maximum and Minimum Values

The y-coordinate of the vertex of the graph of $f(x) = ax^2 + bx + c$ is the **maximum value** of the function when $a < 0$ or the **minimum value** of the function when $a > 0$.

$f(x) = ax^2 + bx + c, a < 0$



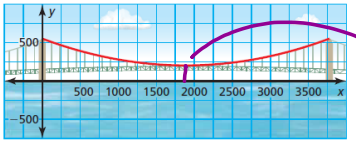
$f(x) = ax^2 + bx + c, a > 0$



Tell whether the function $f(x) = -4x^2 - 24x - 19$ has a minimum value or a maximum value. Then find the value.

The suspension cables between the two towers of the Mackinac Bridge in Michigan form a parabola that can be modeled by

$y = 0.000098x^2 - 0.37x + 552$, where x and y are measured in feet.
What is the height of the cable above the water at its lowest point?



(x, y)

a = .000098

b = −.37

$\left(\frac{-b}{2a}, \frac{-(-.37)}{2(.000098)}\right)$

7 .37 ÷ (2 × .000098

.37 ÷ 2 ÷ .000098

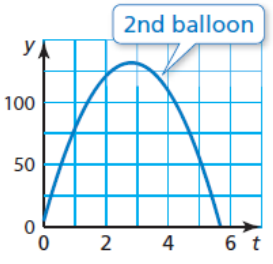
Tell whether the function has a minimum value or a maximum value. Then find the value.

7. $g(x) = 8x^2 - 8x + 6$

8. $h(x) = -\frac{1}{4}x^2 + 3x + 1$

9. The cables between the two towers of the Tacoma Narrows Bridge in Washington form a parabola that can be modeled by $y = 0.00016x^2 - 0.46x + 507$, where x and y are measured in feet. What is the height of the cable above the water at its lowest point?

A group of friends is launching water balloons. The function $f(t) = -16t^2 + 80t + 5$ represents the height (in feet) of the first water balloon t seconds after it is launched. The height of the second water balloon t seconds after it is launched is shown in the graph. Which water balloon went higher?



10. Which balloon is in the air longer? Explain your reasoning.

11. Which balloon reaches its maximum height faster? Explain your reasoning.

Write an equation of a quadratic function that opens up, has a negative y-intercept, and is wider than the graph of $y = x^2$.